Islamic University of Gaza
Engineering Statistics (ECIV 2305)
First Semester (2010-2011)
Midterm Exam

Instructors:
Dr. Ramadan Al-Khatib
Eng. Faten Baalousha

Allowable Time: **80 min**

الشعبة	الرقم الجامعي	اسم الطالب/ة

Closed Book Exam

Question No.	1	2	3	4	Total
Score	7	3	6	7	23
Highest score	7	5	6	7	25

Question 1: (7 points)

A bowl contains seven blue chips and three red chips. Two chips are to be drawn successively at random and without replacement:

(a) Compute the probability that the first draw results in a red chip

Let b = blue

7 blue, 3 red

$(\frac{7}{10})$ b: $\frac{6}{9}$ b, $\frac{3}{9}$ r

$(\frac{3}{10})$ r: $\frac{7}{9}$ b, $\frac{2}{9}$ r

$p(r) = \frac{3}{10}$

(b) If it was known that the first chip is red, what is the probability that the second draw results in a blue chip?

from the Prob tree = $p = \frac{7}{9}$

(c) Compute the probability of the red on the first draw and blue on the second draw.

$p(r, b) = \frac{3}{10} \times \frac{7}{9} = \frac{7}{30}$

(7)

(d) Compute the probability of drawing a red chip on each of the two draws.

$p(r, r) = \frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$

Question 2: (5 points)

Five blue match sticks (أعواد ثقاب), three red match sticks, and two green match sticks are to be arranged on a table. How many different arrangements are possible if:

a) (2 points) The same-color sticks must stay together

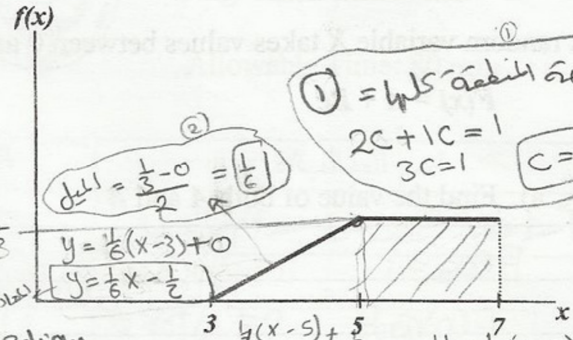
$3! = 3 \times 2 \times 1 = \boxed{6}$

b) (3 points) Only the blue sticks must stand together at the beginning?

Question 3: (6 points)

The pdf of a random variable ***X*** is shown:

$$f(x) = \begin{cases} \frac{1}{6}x - \frac{1}{2} & 3 \le x \le 5 \\ \frac{1}{3} & 5 < x \le 7 \end{cases}$$



$F(x) = \int_3^x \frac{1}{6}y - \frac{1}{2}\,dy = \frac{1}{12}y^2 - \frac{1}{2}y \Big|_3^x$

$F(x) \quad \frac{1}{12}x^2 - \frac{1}{2}x + \frac{3}{4}$ for $3 \le x \le 5$

a) Calculate the median of ***X***

$5 < x \le 7 \rightarrow F(x) = \frac{1}{3} + \int_5^x \frac{1}{3}dy = \frac{3}{4} + \left(\frac{x}{3} - \frac{5}{3}\right)$

$$F(x) = \begin{cases} \frac{1}{12}x^2 - \frac{1}{2}x + \frac{3}{4} & 3 \le x \le 5 \\ \frac{x}{3} - \frac{4}{3} & 5 < x < 7 \end{cases}$$

median $F(x) = \frac{1}{2}$

$\frac{x}{3} - \frac{4}{3} = 0.5 \quad 5 < x < 7$

$x = 5.5 \rightarrow$ median

b) Calculate ***E(X)*** $\quad \int f(x) \cdot x\,dx$

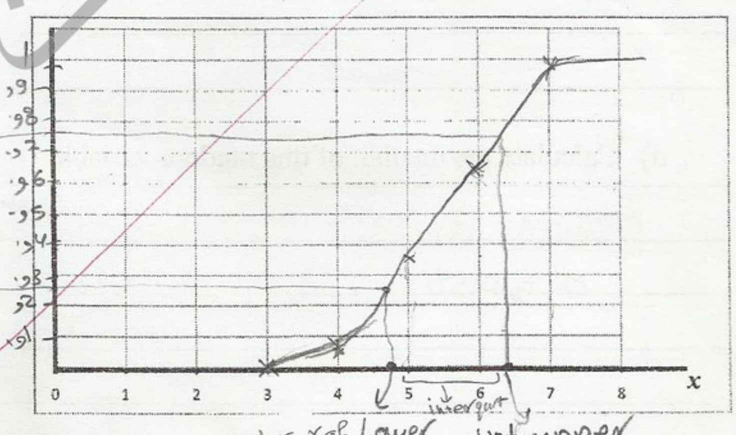
$3 \le x \le 5 \quad E_1(x) = \int_3^5 \left(\frac{1}{6}x - \frac{1}{2}\right) \cdot x\,dx = \int_3^5 \frac{1}{6}x^2 - \frac{1}{2}x\,dx = \frac{x^3}{18} - \frac{x^2}{4}\Big|_3^5 = \frac{25}{36} + \frac{3}{4} = \frac{13}{9}$

$E_2(x) = \int_5^7 x \cdot \left(\frac{1}{3}\right)dx = \frac{x^2}{6}\Big|_5^7 = \frac{49}{6} - \frac{25}{6} = 4$

$E(x) = \frac{13}{9} + 4 = 5.5$

c) Graph the cdf on the chart below

$\frac{3}{4}x$



d) On the cdf graph in part "c", locate the upper quartile and the lower quartile and find their values from the cdf graph upper at x = 0.75 Lower = 0.25

value of x upperr qurtier $\simeq$ 6.25

Value of x Lower qurtiles $\simeq$ 4.75

6

Question 4: (7 points) 7

A random variable X takes values between 0 and ∞ with a cumulative distribution function:

$$F(x) = A + Be^{-x}$$

a) Find the value of both A and B

$F(\infty) = 1$ $1 = A + Be^{-\infty}$ $A = 1$

$F(0) = 0$ $0 = A + Be^{0}$ $B = -1$

$F(x) = 1 - e^{-x}$

b) What is the probability that the random variable X takes a value between 1 and 7

$P = F(7) - F(1)$

$= (1 - e^{-7}) - (1 - e^{-1}) = 0.36696$

c) Find the probability density function

$pdf = \frac{dF_x}{dx} = e^{-x}$ $0 \le x < \infty$

d) Calculate the median of this random variable

median $= F(x) = 0.5$ $0.5 = 1 - e^{-x}$

$e^{-x} = 0.5$ $-x = \ln 0.5$

$x = 0.693$

e) Compute the interquartile range

Upper qurtile $F(x) = 0.75$ $0.75 = 1 - e^{-x} \rightarrow e^{-x} = 0.25$ $x = 1,386$

Lower qurtile $F(x) = 0.25$ $0.25 = 1 - e^{-x} \rightarrow e^{-x} = 0.75$ $x = 0.287$

interqurtile = upperqurtile − Lowerqurtile

$1,386 - 0.287 = 1,098$

Good Luck